

eg. Cindy is going to Alpha Centauri , 4.5 light years away, at  $0.99c$ .

a) what is gamma?

$$= 7.09$$

b) how long does the trip to the star take in Cindy's frame

the distance to Alpha Centauri is Lorentz contracted by

$$L = L_0 / \gamma = 4.5 / 7.09 = 0.6347 \text{ light years}$$

$$t = d/v = 0.6347 / 0.99 = 0.6411 \text{ years}$$

$$= 0.64 \text{ years}$$

ii) Amy (stays on Earth)'s frame

$$t = d/v = 4.5 \text{ light years} / 0.99 \times \text{speed of light}$$

$$= 4.5 / 0.99 = 4.5455 \text{ years}$$

c) What time is it on Earth in Cindy's frame before/after the turn around. (tricky)

$$t' = \gamma(t - vx/c^2)$$

$$t' = 7.09 \times (0.6411 - (0.99 \times 0.6347)) = 0.0904 \text{ years}$$

$$7.09 \times (0.6411 + (0.99 \times 0.6347)) = 9.0004 \text{ years}$$

so, before the turnaround, Cindy sees time on Earth as going slower, on 0.09 years while she experiences 0.64 years.

After she turns around, she "sees" time on Earth as being 9.0 years. So when she gets back to

Earth, she has only experienced  $2 \times 0.6411 = 1.2822$  years while Amy experiences  $9.0004 + (0.6411/7.09) = 9.0908$  years.  
 $4.5455 \times 2 = 9.091$  alternately

d) When Cindy gets back, how much younger is she than Amy?

$9.0004 - 1.2822 = 7.7182$  years younger

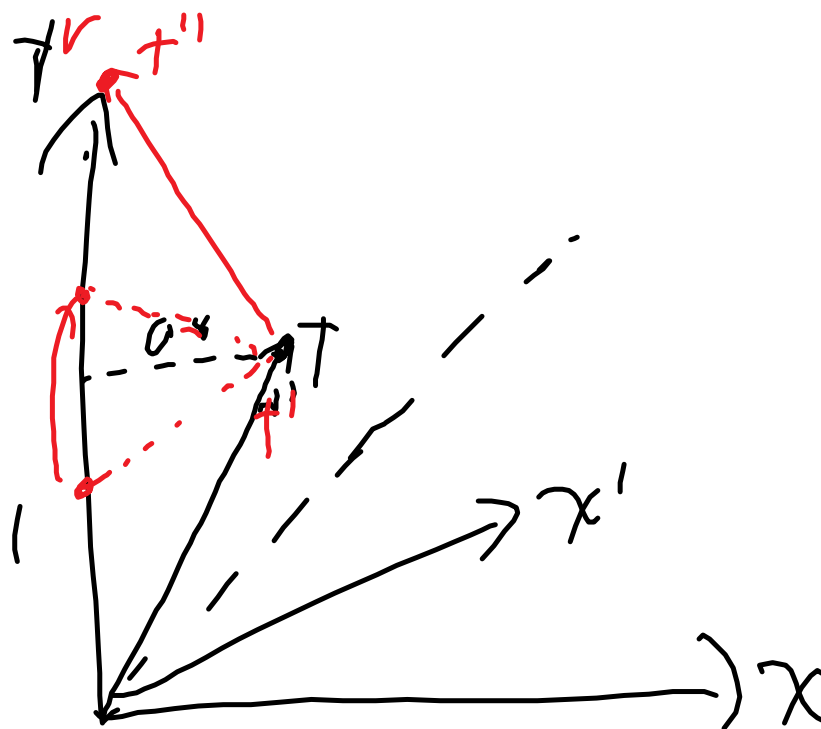
e) What distance was covered in Cindy's frame?

$L = L_0/\gamma = 4.5/7.09 = 0.6347$  light years

$0.6347 \times 2 = 1.2694$  light years

Homework - look over the exam style questions.

Q3, 4, 6 (8 is HL only)



a) Two twins, one travels near  $c$  away and

back. Each twin sees the other's clock running slow. How can both times be slower?

b) Resolved because

- i) the travelling twin's sees the distance length contracted, so it takes them less time, so they come back younger.
- ii) when the twin turns around, they shift spacetimes, so the time on Earth shifts according to

$$t' = \gamma(t - vx/c^2)$$

as  $v$  changes from  $+$  to  $-$ ,  $t'$  changes dramatically - so time on Earth changes in the frame of the travelling twin

c) i -  $t = d/v = 12 \text{ light years} / 0.80c = 15 \text{ years}$

ii) Lorentz contracted

$\gamma =$

$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.64 \cancel{c^2} / \cancel{c^2}}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6} = \frac{5}{3}$$

$$\gamma = 1.666 \dots$$

v

$$\gamma = 1.666 \dots$$

$$12/1.66666667 = 7.19999999$$

$$7.1999999/0.8 = 8.9999999 = 9.0 \text{ years}$$

iii) A

$$t' = \gamma(t - vx/c^2) = 1.66667 \times (8.999 - (0.8 \times 7.1999)) = 5.39848 \text{ years}$$

$$\text{iv } 1.66667 \times (8.999 + (0.8 \times 7.1999)) = 24.59825 \text{ years}$$

v) 30 years (15x2)

vi) 18 years (9x2)

On earth twin aged 30 years, the travelling twin aged 18 years.

You send a signal down a cable with  $3.0 \times 10^{-3} \text{ W/m}^2$  of intensity. 2.0 km along the cable, the signal has dropped to  $2.0 \times 10^{-3} \text{ W/m}^2$ . What is the attenuation per unit distance?

$$\text{attenuation} = 10 \log I/I_0$$

$$\text{attenuation} =$$

$$10 \times \log(2/3) = -4.054651081081644 = -4.05465$$

1081081644

$$\text{attenuation/d} = -4.05463/2 = -2.02731 \text{ dB/km}$$

What would be the signal left after another 5.0km with no boost?

$$-2.02731 \text{ dB/km} \times 5.0 \text{ km}$$

$$-2.02731 \times 5 = -10.13655 \text{ dB}$$

$$\text{attenuation} = 10 \log(I/I_0)$$

$$-10.13655 = -10 \log(I/2.0 \times 10^{-3} \text{ W/m}^2)$$

$$-10.13655/10 = -1.01365$$

$$10^{-1.01365} = 0.09691 = (I/2.0 \times 10^{-3} \text{ W/m}^2)$$

$$I = 0.09691 \times 2 = 0.19382$$

$$1.9 \times 10^{-4} \text{ W/m}^2$$